Method for optimizing the correction of the eye's higher-order aberrations in the presence of decentrations

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The use of a correcting element to compensate for higher-order aberrations in an optical system often requires accurate alignment of the correcting element. This is not always possible, as in the case of a contact lens on the eye. We propose a method consisting of partial correction of every aberration term to minimize the average variance of the residual wave-front aberration produced by Gaussian decentrations (translations and rotations). Analytical expressions to estimate the fraction of every aberration term that should be corrected for a given amount of decentration are derived. To demonstrate the application of this method, three examples are used to compare performance with total and with partial correction. The partial correction is more robust and always yields some benefit regardless of the amount of decentration. © 2002 Optical Society of America

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Experiments have demonstrated that visual performance enhanced by adaptive optics can be better than standard performance, because the eye's higher-order aberrations beyond defocus and astigmatism are corrected.[1] Encouraged by these findings, researchers are engaged in an effort to implement improved ideal correcting methods such as customized contact lenses or customized laser refractive surgery to improve vision. However, while closed-loop adaptive optics can track changes in the wave front, the performance of any static correcting method is limited by residual aberrations arising in particular from decentrations.[2,3] We have shown that the effect of decentration is different for each particular aberration corrected.[2] Thus we introduced the concept of selective correction and proposed a binary rule to select those aberrations the correction of which still yields a benefit in the presence of decentrations.[2] The rule prescribed that a particular aberration be left uncorrected if decentration exceeds a certain value; otherwise, the correction will produce larger aberrations. In this Communication, we present a more sophisticated method for optimizing the correction of the aberrations of the eye with a decentered correcting element. The method consists of correcting a fraction of every aberration term rather than leaving some terms uncorrected.

Let the wave-front aberration (WA) of an optical system (OS), in particular the eye, be described as a linear combination of Zernike polynomials $Z_n^{\pm m}$ with coefficients $a_n^{\pm m}$, where n is the radial order and m is the angular order.[4] We propose a method to correct only an appropriate fraction $\gamma_n^{\pm m}$ of every aberration term for a given decentration. The Zernike coefficients for the correcting method (CM), e.g., a contact lens, will be

$$b_n^{\pm m} = -\gamma_n^{\pm m} a_n^{\pm m}. \tag{1}$$

With no decentration, the residual WA after correction would be

$$\mathrm{WA_{residual}} = \mathrm{WA_{OS}} + \mathrm{WA_{CM}} = \sum_{n,\pm m} (1 - \gamma_n^{\pm m}) a_n^{\pm m} Z_n^{\pm m}. \tag{2}$$

It is clear that with a perfect centration, $\gamma_n^{\pm m}$ should be equal to 1. Our goal is to calculate the fraction $\gamma_n^{\pm m}$ in order to have an optimum correction (i.e., minimum residual aberrations) in the presence of decentrations. Figure 1 shows a diagram illustrating the method. After decentration (translation/rotation), the WA of the correcting method can be obtained as a translated and rotated version of the original WA. With a change of coordinates, the decentered WA for the correcting method can be reexpressed in Zernike polynomials with coefficients

$$C_n^{\pm m} = -([T][R](\gamma a))_n^{\pm m}, \tag{3}$$

where $[T]$ and $[R]$ are matrices for rotation and translation, respectively.[2] Thus the residual WA is

$$\mathrm{WA_{residual}} = \mathrm{WA_{OS}} + \mathrm{WA_{CM}}\ \text{(decentered)}$$
$$= \sum_{n,\pm m} (a_n^{\pm m} + C_n^{\pm m}) Z_n^{\pm m}, \tag{4}$$

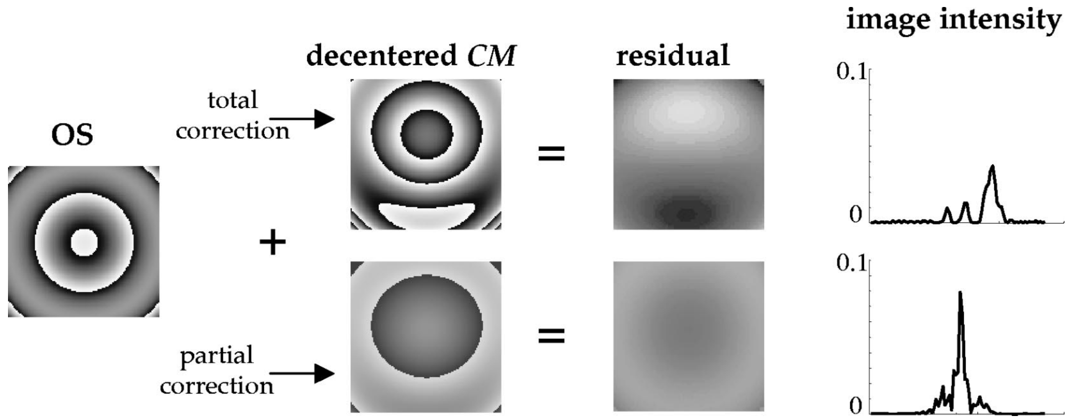

Fig. 1. Illustration of the concept of the method. An optical system (OS) with pure spherical aberration is corrected by a correcting method (CM) that is decentered with respect to the pupil. When the total aberration of the OS is corrected, the residual aberrations are larger than those corresponding to the correction of only a fraction of the original aberration, as shown by the image intensity profiles. In the presence of decentration, the correction of spherical aberration produces coma that is proportional to the decentration and to the amount of spherical aberration corrected; with partial correction, a residual spherical aberration is uncorrected, but the amount of coma produced is smaller, giving minimum variance for the total aberrations.

Table 1. Values of the Factor F_n^m

Order		
n	m	F_n^m
3	1	48
3	3	24
4	0	80
4	2	80
4	4	40
5	1	192
5	3	156
5	5	60
6	0	280
6	2	280
6	4	224
6	6	84

and the variance of the residual WA will be

$$\mathrm{var} = \sum_{n,\pm m} (a_n^{\pm m} + C_n^{\pm m})^2. \tag{5}$$

Since decentration is expected to be a dynamic process (vibration or motion of the optical elements), we are interested in the average performance within intervals of decentration. In particular, a contact lens translates and rotates on the eye. We model the positions of the correcting method by means of a Gaussian distribution with standard deviations σ_t, σ_r for translation and rotation, respectively. The average variance of the residual WA, $\langle \mathrm{var} \rangle_{\sigma_t,\sigma_r}$ is calculated as the integral of the variances for each decentration weighted by the Gaussian distribution. The average variance can be minimized to obtain the optimum value of the fraction $\gamma_n^{\pm m}$. By taking the derivatives of $\langle \mathrm{var} \rangle_{\sigma_t,\sigma_r}$ with respect to $\gamma_n^{\pm m}$ and setting equal to zero, we obtain

$$\gamma_n^{\pm m}(\sigma_t, \sigma_r) = \frac{\exp(-m^2\sigma_r^2/2)}{1 + F_n^m \sigma_t^2/r_o^2}, \tag{6}$$

where r_o is the pupil radius and F_n^m is a factor depending on n and m (values are listed in Table 1). The fraction $\gamma_n^{\pm m}$ is 1 for no decentration and tends asymptotically to 0 when decentration increases.

In Fig. 2 we show three examples of results comparing the performance of partial correction and total correction. The coefficients of the WA to be corrected are normalized to have a variance equal to 1. The figures show the average variance of the residual WA, as a function of the interval of decentration, when the aberration is totally corrected (thin solid curve) and when only the fraction $\gamma_n^{\pm m}$ of the term is corrected (thick solid curve). The fraction $\gamma_n^{\pm m}$ is also shown (dotted curve). A variance below 1 indicates a benefit of correcting the aberrations, a variance of 1 indicates no benefit, and a value higher than 1 means a detriment. Figure 2(a) shows the result of correcting spherical aberration (Z_4^0), as a function of the standard deviation of translation. For instance, for $\sigma_t/r_o = 0.11$ the total correction of the aberration yields no benefit. However there is still a 50% benefit if only a fraction of $\sim$0.5 of the term is corrected instead. While the total correction may introduce more aberrations than those of the uncorrected system [see $\sigma_t/r_o > 0.11$ in Fig. 2(a)], the partial correction always yields some benefit. Figure 2(b) shows the correction of a term Z_6^6, as a function of the standard deviation of rotation. Figure 2(c) shows a corrected WA consisting of two third-order aberrations, Z_3^1 and Z_3^3, for different standard deviations of translation. The fraction of Z_3^1 corrected is shown by the dotted curve and the fraction of Z_3^3 by the dashed curve.

In summary, we have proposed a method to correct an optimum fraction of the aberrations of the eye and obtain the minimum residual aberrations considering the effects of Gaussian decentrations of the correcting element (e.g., a contact lens). Calculations show how, with increasing decentration, the correction of every kind of aberration should be gradually reduced according to the fraction in

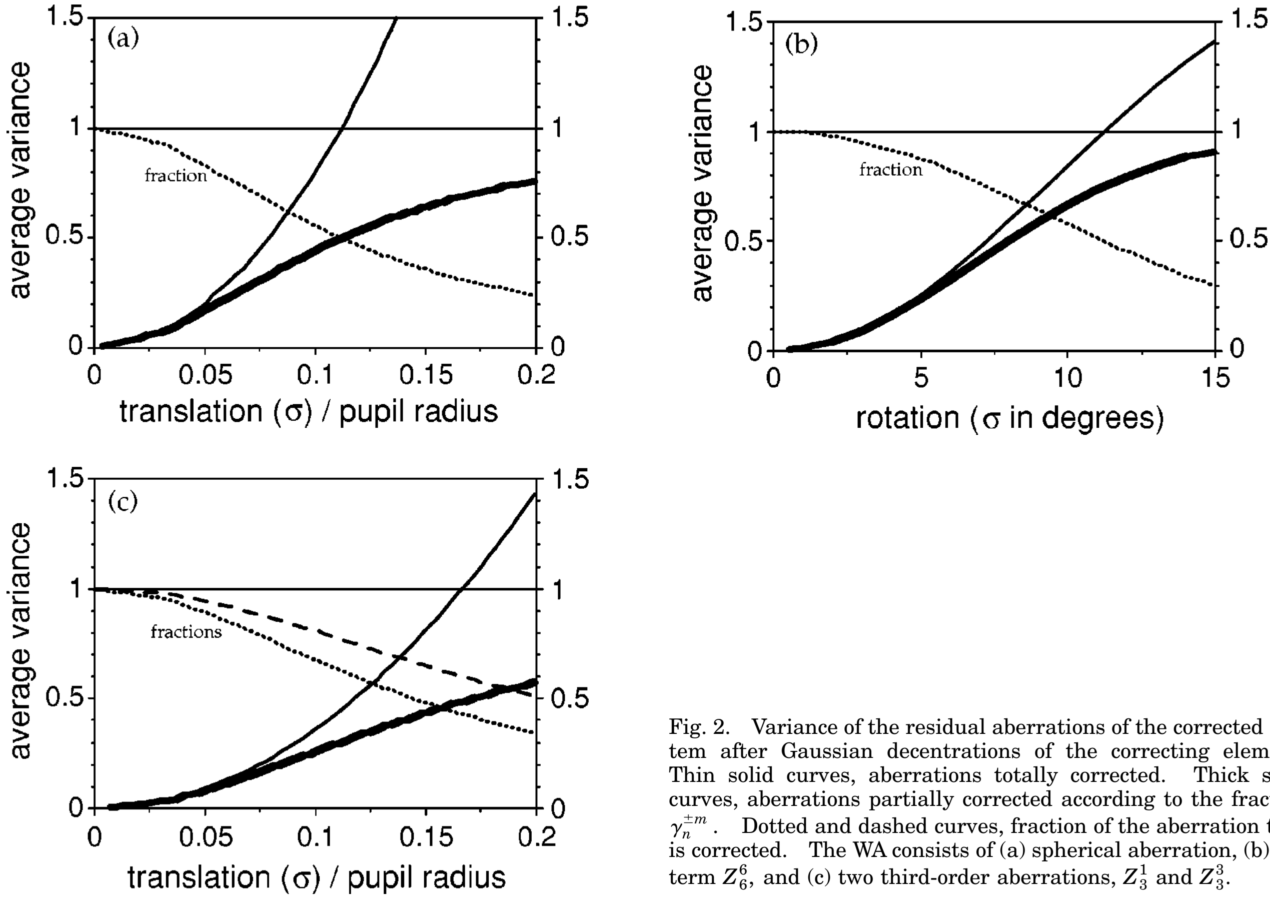

Fig. 2. Variance of the residual aberrations of the corrected system after Gaussian decentrations of the correcting element. Thin solid curves, aberrations totally corrected. Thick solid curves, aberrations partially corrected according to the fraction $\gamma_n^{\pm m}$. Dotted and dashed curves, fraction of the aberration that is corrected. The WA consists of (a) spherical aberration, (b) the term Z_6^6, and (c) two third-order aberrations, Z_3^1 and Z_3^3.

Eq. (6). The method is general and can be applied to correct the aberrations of optical systems other than the eye.

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