Effect of rotation and translation on the expected benefit of an ideal method to correct the eye's higher-order aberrations

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Received June 16, 2000; revised manuscript received November 13, 2000; accepted November 14, 2000

An ideal correcting method, such as a customized contact lens, laser refractive surgery, or adaptive optics, that corrects higher-order aberrations as well as defocus and astigmatism could improve vision. The benefit achieved with this ideal method will be limited by decentration. To estimate the significance of this potential limitation we studied the effect on image quality expected when an ideal correcting method translates or rotates with respect to the eye's pupil. Actual wave aberrations were obtained from ten human eyes for a 7.3-mm pupil with a Shack–Hartmann sensor. We computed the residual aberrations that appear as a result of translation or rotation of an otherwise ideal correction. The model is valid for adaptive optics, contact lenses, and phase plates, but it constitutes only a first approximation to the laser refractive surgery case where tissue removal occurs. Calculations suggest that the typical decentrations will reduce only slightly the optical benefits expected from an ideal correcting method. For typical decentrations the ideal correcting method offers a benefit in modulation 2–4 times higher (1.5–2 times in white light) than with a standard correction of defocus and astigmatism. We obtained analytical expressions that show the impact of translation and rotation on individual Zernike terms. These calculations also reveal which aberrations are most beneficial to correct. We provided practical rules to implement a selective correction depending on the amount of decentration. An experimental study was performed with an aberrated artificial eye corrected with an adaptive optics system, validating the theoretical predictions. The results in a keratoconic subject, also corrected with adaptive optics, showed that important benefits are obtained despite decentrations in highly aberrated eyes.

OCIS codes: 330.7310, 330.4460, 330.5370, 220.1000.

1. INTRODUCTION

The optical quality of the eye imposes the first limit on spatial vision.[1] Improving the optics of the eye will improve vision up to the point where the optical quality exceeds the limits set by the spacing between cones and subsequent neural factors. It has long been known that the human eye is far from a perfect optical system.[2] Although spectacles and contact lenses have been successfully used to correct defocus and astigmatism, research with several techniques[3–14] has shown that the eye suffers from higher-order aberrations, besides defocus and astigmatism, that also degrade retinal image quality. In daylight conditions, the natural pupil is an optimum size (~3 mm) for balancing the effects of diffraction and aberrations in normal young subjects.[4] Aberrations for small pupils are dominated by the second-order and, thus a conventional correction offers a sufficient improvement. However, it has been found that the higher-order aberrations have a significant impact on the retinal image quality in normal eyes for large pupils,[10,15] and also for older subjects[16] as well as in abnormal subjects (postrefractive surgery or keratoconic patients, for instance[17–19]). In 1962 Smirnov[3] suggested that it would be possible to manufacture customized lenses to compensate for the higher-order aberrations of individual eyes. Recent developments increase the probability that Smirnov's suggestion may be realized: more rapid and accurate instruments for measuring the ocular aberrations,[10,14,20] a better knowledge of the aberrations in the human population,[21] and new techniques to correct the higher-order aberrations. Thus lathe technology allows the manufacture of contact lenses with nearly any aberration profile, and photosculptured phase plates have been made with the aberration pattern of the eye.[22] There is an ongoing effort to refine laser refractive surgery to the point that it can correct other defects besides conventional refractive errors.[23,24]

By using an adaptive optics (AO) system, Liang *et al.*[25] successfully corrected higher-order aberrations and provided normal eyes with supernormal optical quality. For a pupil of 6 mm they found improvements in modulation transfer function and contrast sensitivity up to sixfold at

27.5 cycles per degree (c/deg) after correction. Also, through adaptive optics subjects could resolve fine gratings that were invisible under normal viewing conditions. More recently, similar though smaller improvements in contrast sensitivity and also visual acuity have been obtained when extending the study to broadband illumination.[26] These results encourage the implementation of supercorrecting procedures, such as customized contact lenses or laser refractive surgery, to compensate for the higher-order aberrations as well as defocus and astigmatism, which could improve the eye's optical performance. However, a potential limitation is that the expected improvement in vision from these ideal correcting procedures will be reduced by decentrations relative to the pupil. Studies have shown that vision with contact lenses is essentially dynamic. In particular, movement of the lens as a result of blinking[27] or as a result of the eye's downward rotation[28] may be expected to affect vision. On the other hand, the eye's motion during laser refractive surgery[29] may reduce the accuracy of a customized ablation. For an ideal correcting method to be effective either it must be designed to reduce movement[30,23] or its optical performance must be relatively unaffected by typical decentrations.

In this paper we study the effect on image quality expected when an ideal correcting method translates or rotates with respect to the eye, based on the wave-aberration data of populations of ten eyes. We discuss ranges of tolerance in both monochromatic and white light and compare the benefit of the ideal correction with the benefit that a conventional correction provides. We also present equations that offer insight into which aberrations are more beneficial to correct in the presence of decentrations. To support the theoretical results, we carried out an experiment in which the aberrations were measured when an artificial eye was decentered with respect to an adaptive optics system designed to correct the artificial eye's higher-order aberrations. The eye's aberrations of a keratoconic subject were also corrected with the adaptive optics system, and the loss of compensation was measured after applying decentrations.

2. METHODS

A. Wave-Aberration Data

We measured the wave aberration (WA) in ten normal human eyes using a Shack–Hartmann wave-front sensor (see Ref. 10 for details of the system). Measurements

Table 1. Zernike Polynomials up to Sixth Order

n	$\pm m$	Zernike Polynomials	Monomial Representation
0	0	1	1
1	1	$2\rho\cos\theta$	x
	−1	$2\rho\sin\theta$	y
2	0	$\sqrt{3}(2\rho^2 - 1)$	$-1 + 2(x^2 + y^2)$
	2	$\sqrt{6}\rho^2\cos 2\theta$	$x^2 - y^2$
	−2	$\sqrt{6}\rho^2\sin 2\theta$	$2xy$
3	1	$\sqrt{8}(3\rho^3 - 2\rho)\cos\theta$	$x[3(x^2 + y^2) - 2]$
	−1	$\sqrt{8}(3\rho^3 - 2\rho)\sin\theta$	$y[3(x^2 + y^2) - 2]$
	3	$\sqrt{8}\rho^3\cos 3\theta$	$x(x^2 - 3y^2)$
	−3	$\sqrt{8}\rho^3\sin 3\theta$	$y(3x^2 - y^2)$
4	0	$\sqrt{5}(6\rho^4 - 6\rho^2 + 1)$	$6(x^2 + y^2)^2 - 6(x^2 + y^2) + 1$
	2	$\sqrt{10}(4\rho^4 - 3\rho^2)\cos 2\theta$	$(x^2 - y^2)[4(x^2 + y^2) - 3]$
	−2	$\sqrt{10}(4\rho^4 - 3\rho^4)\sin 2\theta$	$2xy[4(x^2 + y^2) - 3]$
	4	$\sqrt{10}\rho^4\cos 4\theta$	$x^4 - 6x^2y^2 + y^4$
	−4	$\sqrt{10}\rho^4\sin 4\theta$	$4xy(x^2 - y^2)$
5	1	$\sqrt{12}(10\rho^5 - 12\rho^3 + 3\rho)\cos\theta$	$x[10(x^2 + y^2)^2 - 12(x^2 + y^2) + 3]$
	−1	$\sqrt{12}(10\rho^5 - 12\rho^3 + 3\rho)\sin\theta$	$y[10(x^2 + y^2)^2 - 12(x^2 + y^2) + 3]$
	3	$\sqrt{12}(5\rho^5 - 4\rho^3)\cos 3\theta$	$x(x^2 - 3y^2)[5(x^2 + y^2) - 4]$
	−3	$\sqrt{12}(5\rho^5 - 4\rho^3)\sin 3\theta$	$y(3x^2 - y^2)[5(x^2 + y^2) - 4]$
	5	$\sqrt{12}\rho^5\cos 5\theta$	$16x^5 - 20x^3(x^2 + y^2) + 5x(x^2 + y^2)^2$
	−5	$\sqrt{12}\rho^4\sin 5\theta$	$16y^5 - 20y^3(x^2 + y^2) + 5y(x^2 + y^2)^2$
6	0	$\sqrt{7}(20\rho^6 - 30\rho^4 + 12\rho^2 - 1)$	$20(x^2 + y^2)^3 - 30(x^2 + y^2)^2 + 12(x^2 + y^2) - 1$
	2	$\sqrt{14}(15\rho^6 - 20\rho^4 + 6\rho^2)\cos 2\theta$	$(x^2 - y^2)[15(x^2 + y^2)^2 - 20(x^2 + y^2) + 6]$
	−2	$\sqrt{14}(15\rho^6 - 20\rho^4 + 6\rho^2)\sin 2\theta$	$2xy[15(x^2 + y^2)^2 - 20(x^2 + y^2) + 6]$
	4	$\sqrt{14}(6\rho^6 - 5\rho^4)\cos 4\theta$	$(x^4 + y^4 - 6x^2y^2)[6(x^2 + y^2) - 5]$
	−4	$\sqrt{14}(6\rho^6 - 5\rho^4)\sin 4\theta$	$4xy(x^2 - y^2)[6(x^2 + y^2) - 5]$
	6	$\sqrt{14}\rho^6\cos 6\theta$	$32x^6 - 48x^4(x^2 + y^2) + 18x^2(x^2 + y^2)^2 - 1$
	−6	$\sqrt{14}\rho^6\sin 6\theta$	$2xy[16x^4 - 16x^2(x^2 + y^2) + 3(x^2 + y^2)^2]$

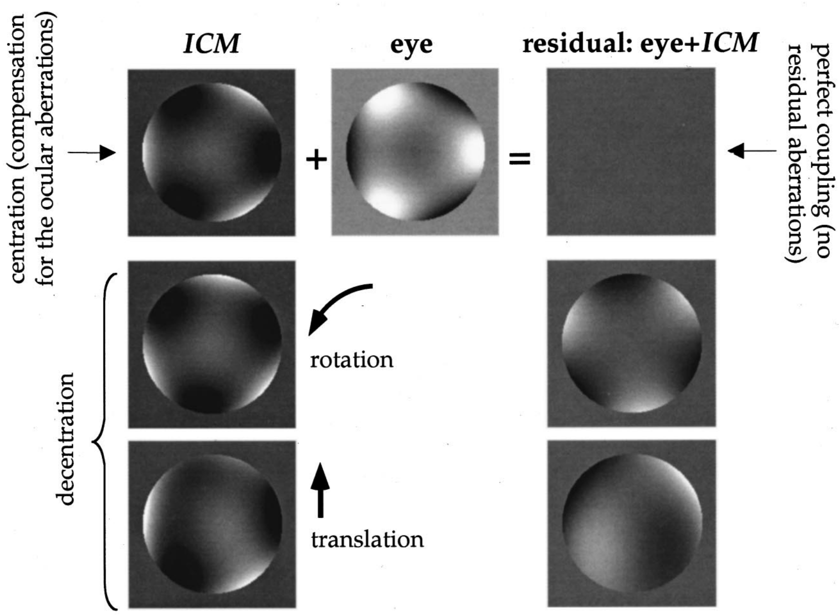

Fig. 1. Example of wave-aberration maps of the ideal correcting method, the eye, and the coupling. The correcting method is ideal in the sense that it corrects the monochromatic aberrations when it is centered. Decentration produces a mismatch.

were made for a pupil 7.3 mm in diameter after dilation with tropicamide. Subjects had a defocus between −1.5 and 1.5 diopters (D) (mean ± standard deviation = 0 ± 0.8 D), an astigmatism from 0 to −1.5 D (−0.5 ± 0.5 D), and normal visual acuity. Age ranged from 21 to 38 years old. WA's were described as a combination of Zernike polynomials[31,32] ($Z_n^{\pm m}$) up to sixth order (see Table 1) over a truncated pupil of 6 mm with coefficients ($a_n^{\pm m}$):

$$\mathrm{WA_{eye}}(\rho, \theta) = \sum_{n,\pm m} a_n^{\pm m} Z_n^{\pm m}(\rho, \theta), \qquad n = 1, 2, \ldots, 6. \tag{1}$$

The mean value and standard deviation across the ten eyes of the rms of the WA for a 6-mm pupil was 0.97 ± 0.47 micrometers, and 0.33 ± 0.16 micrometers without including defocus and astigmatism.[33]

B. Wave Aberration for the Ideal Correcting Method and Residual Wave Aberration after Decentration

For each eye, we considered an ideal correcting method (ICM) that would completely correct the eye's WA when centered:

$$\mathrm{WA_{ICM}} = -\sum_{n,\pm m} a_n^{\pm m} Z_n^{\pm m}, \tag{2}$$

and the residual WA when the correction is applied with a translation and rotation with respect to the pupil of the eye (Fig. 1):

$$\begin{aligned}\mathrm{WA_{residual}} &= \mathrm{WA_{eye}} + \mathrm{WA_{ICM(decentered)}}\\ &= \sum_{n,\pm m} a_n^{\pm m} Z_n^{\pm m} - \sum_{n,\pm m} C_n^{\pm m} Z_n^{\pm m}.\end{aligned} \tag{3}$$

Zernike coefficients $C_n^{\pm m}$ represent the decentered version of the WA of the correcting method. To obtain this WA, we took a circular section from the original WA at a translated center and then rotated it (Fig. 2). We considered that the ICM corrects for the entire region of the pupil sampled (7.3 mm). Since the correction extends beyond the pupil studied (6 mm), when the ICM is displaced there is still an overlapping between the eye's WA and the ICM's WA as shown in Fig. 2, and therefore there is still a partial compensation.

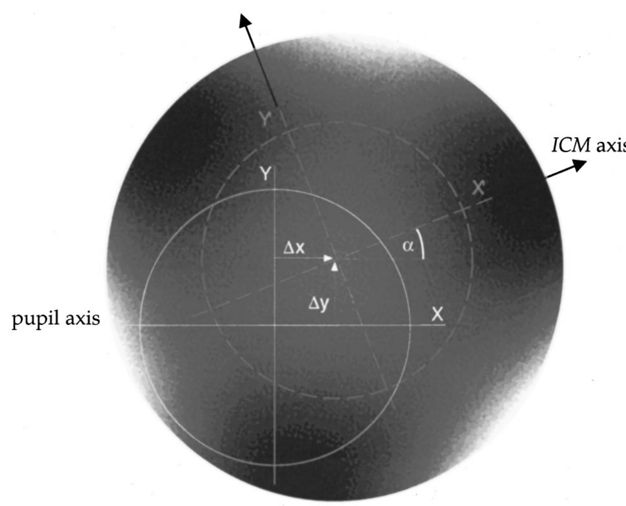

Fig. 2. Wave aberration of the correcting method (axis $x'y'$) decentered with respect to the eye (axis xy). The wave aberration added to that of the eye is the rotated (angle α) version and the translated (Δx, Δy) version of the wave aberration of the centered correcting method. The correcting method extends beyond the eye's pupil.

The correcting method was assumed to be conjugate with the eye's pupil. In the case of a contact lens, small tilts and axial displacements[34] due to the translation of the lens on the cornea were computed to have a minor impact on the residual WA. Moreover, the axial displacement is a negligible factor in comparison with translation and rotation.[35]

By introducing the change of coordinates

$$\begin{cases} x' = (x - \Delta x)\cos\alpha + (y - \Delta y)\sin\alpha, \\ y' = (y - \Delta y)\cos\alpha - (x - \Delta x)\sin\alpha, \end{cases} \tag{4}$$

where α is angle of rotation and $\Delta x, \Delta y$ are translations along the X and the Y axis, respectively, into the monomial representation of each Zernike polynomial (see Table 1) and rearranging terms, we reexpressed the WA of the decentered correcting method in a Zernike expansion. The coefficients (C_i) for the new expansion can be obtained analytically from the original coefficients (a_k) by means of the following matrix product:

$$C_i = \sum_{j,k} T_{ij} R_{jk} a_k\,, \tag{5}$$

where [T] and [R] are matrices for rotation and translation, respectively. Thus the coefficients for the residual WA are

$$(1 - T_{ij} R_{jk}) a_k\,. \tag{6}$$

We also computed the residual WA considering that the ICM corrected only a certain set of aberration terms instead of all of them. In those cases, the ocular WA corresponding to the uncorrected terms was added to the residual WA:

$$\mathrm{WA}_{\text{residual}} = \sum_{\text{noncorrected}} a_i Z_i + \sum_{\text{corrected}} (1 - T_{ij} R_{jk}) a_k Z_i\,. \tag{7}$$

C. Matrices for Rotation and Translation

Owing to the property of invariance under rotation[31] of the Zernike polynomials, the matrix [R] can be easily constructed by adding submatrices of the type

$$\begin{bmatrix} \cos m\alpha & -\sin m\alpha \\ \sin m\alpha & \cos m\alpha \end{bmatrix} \tag{8}$$

for transforming the coefficients $a_n^{\pm m}$ into coefficients $C_n^{\pm m}$ for $m \neq 0$ and setting equal to 1 the matrix elements corresponding to $a_n^{m=0}$.

The matrix [T] for converting the WA of the ideal correcting method after a translation[36] is shown in Table 2. $A = \Delta x / r_0$ and $B = \Delta y / r_0$ are the values of translation along the horizontal and vertical directions, respectively, normalized to the pupil radius (r_0). Each matrix element must be scaled by the factor $[(n + 1)(n' + 1)]^{1/2}$, where n' means order corrected and n means order affected. From the symmetry of the matrix the reader can easily extend the results to the higher orders. For simplicity, only the linear matrix elements depending on A and B are shown in Table 2. They were found to represent an accurate approximation for values of translation lower than 0.15 times the pupil radius. However, the higher powers should be progressively considered as the amount of translation increases. In general, an aberration of order n' introduces with translation the next additional aberrations:

1. Orders $n' - 1, n' - 3, n' - 5, \ldots,$ depending on A and B;

Table 2. Conversion Matrix for Contact-Lens Aberration Coefficients after a Translation

		n' (order corrected)																										
		1		2			3				4					5						6						
n		a_1^1	a_1^{-1}	a_2^0	a_2^2	a_2^{-2}	a_3^1	a_3^{-1}	a_3^3	a_3^{-3}	a_4^0	a_4^2	a_4^{-2}	a_4^4	a_4^{-4}	a_5^1	a_5^{-1}	a_5^{-3}	a_5^{-3}	a_5^5	a_5^{-5}	a_6^0	a_6^2	a_6^{-2}	a_6^4	a_6^{-4}	a_6^6	a_6^{-6}
	C^{1}_{1}	1		-√2 A	-A	-B					-√2 A	-A	-B									-√2 A	-A	-B				
1	C^{-1}_{1}		1	-√2 B	B	-A					-√2 B	B	-A									-√2 B	B	-A				
	C^{0}_{2}			1			-√2 A	-√2 B								-√2 A	-√2 B											
	C^{2}_{2}				1		-A	B	-A	-B						-A	B	-A	-B									
2	C^{-2}_{2}					1	-B	-A	B	-A						-B	-A	B	-A									
	C^{1}_{3}						1				-√2 A	-A	-B									-√2 A	-A	-B				
	C^{-1}_{3}							1			-√2 B	B	-A									-√2 B	B	-A				
	C^{3}_{3}								1			-A	B	-A	-B								-A	B	-A	-B		
3	C^{-3}_{3}									1		-B	-A	B	-A								-B	-A	B	-A		
	C^{0}_{4}										1					-√2 A	-√2 B											
	C^{2}_{4}											1				-A	B	-A	-B									
	C^{-2}_{4}												1			-B	-A	B	-A									
	C^{4}_{4}													1				-A	B	-A	-B							
4	C^{-4}_{4}														1			-B	-A	B	-A							
	C^{1}_{5}															1						-√2 A	-A	-B				
	C^{-1}_{5}																1					-√2 B	B	-A				
	C^{3}_{5}																	1					-A	B	-A	-B		
	C^{-3}_{5}																		1				-B	-A	B	-A		
	C^{5}_{5}																			1					-A	B	-A	-B
5	C^{-5}_{5}																				1				-B	-A	B	-A
	C^{0}_{6}																					1						
	C^{2}_{6}																						1					
	C^{-2}_{6}																							1				
	C^{4}_{6}																								1			
	C^{-4}_{6}																									1		
	C^{6}_{6}																										1	
6	C^{-6}_{6}																											1

2. Orders $n' - 2, n' - 4, n' - 6, \ldots,$ depending on A^2, B^2, AB;
3. Orders $n' - 3, n' - 5, \ldots,$ depending on the third powers of A and B; and so on.

For instance, in the case of dependence on A and B, coma generates astigmatism and defocus, spherical aberration produces coma and tilt, and defocus or astigmatism produces only tilt (i.e., for a conventional lens that corrects astigmatism and defocus, translation produces only a shift in the retinal image).

D. Image Quality

From the residual WA's, the previous removal of the piston, and the tilt terms, we computed the modulation transfer function[37] (MTF) to estimate the retinal image quality in each eye after correction with the ideal correcting method as a function of rotation and translation. The MTF was obtained in monochromatic light for 555 nm. We also computed the MTF in white light, including axial and lateral chromatic aberration.[38,39] We computed the point-spread functions between 400 and 700 nm at 10-nm intervals, assuming an equal-energy spectrum, weighted by the photopic spectral sensitivity curve for the CIE observer, and superposed with the shift corresponding to the lateral chromatic aberration.[39] In the case of a contact lens, in white light the tilt when the lens translates on the cornea produces additional chromatic aberration, which was included in the calculations.

We also computed the average MTF considering a distribution of movement (translation/rotation) of the correcting method. This average MTF within an interval of movement was calculated as the sum of the MTF for each decentration weighted by a Gaussian distribution:

$$\frac{1}{(2\pi)^{3/2}\sigma_t^2\sigma_r}\exp(-(\Delta x^2 + \Delta y^2)/2\sigma_t^2)\exp(-\alpha^2/2\sigma_r^2), \quad (9)$$

where σ_t, σ_r are the widths of the Gaussian distribution of the translation and rotation, respectively. The average MTF calculated in this way gives us an estimation of the average image quality for dynamic decentrations.

The MTF averaged across the ten eyes was obtained in every case and was compared with that resulting from a conventional correction that corrected only defocus and astigmatism instead of all the orders. The quantitative evaluation of the visual improvement provided by the ideal correcting method rests on the calculation of the eye's MTF when only defocus and astigmatism are corrected. Owing to the higher-order aberrations, nonzero values of defocus and astigmatism can improve subjective image quality. We applied a computational method that would approximate the process of subjectively refracting a real eye. The method consisted of searching the parameter space corresponding to all three Zernike coefficients for defocus and astigmatism to optimize a metric defined as the volume of the contrast sensitivity function (CSF). This CSF was calculated as the product of the MTF and the neural CSF achieved by interferometry.[40]

3. SELECTION RULES FOR CORRECTING SET OF TERMS

The sensitivity to translation and rotation is different for different aberration terms. This suggests the idea of leaving uncorrected the terms less tolerant to decentration. In this section we propose rules to decide which terms should be left uncorrected because their correction produces a loss instead of a benefit.

A. Fixed Decentrations

With use of Eq. (6) the variance of the residual WA after a rotation results:

$$\mathrm{rms}^2 = 4\sum_{n,m\neq 0}[(a_n^{+m})^2 + (a_n^{-m})^2]\cdot \sin^2 m\alpha/2. \quad (10)$$

It follows from Eq. (10) that the correction of the two terms $Z_n^{\pm m}$ produces after a rotation a contribution to the variance of $4\sin^2 m\alpha/2$ times the contribution if those terms were left uncorrected. This implies that the higher-order corrections (including terms with higher angular orders) are less tolerant to rotation. The correction of a third-order coma aberration ($m = 1$) has the largest tolerance. The correction of astigmatism ($m = 2$) offers a benefit when the rotation is lower than 30°, or the triangular astigmatism ($m = 3$) offers a benefit when the angle does not reach 20°, and so on. Obviously the rotation has no effect on the correction of the rotationally symmetric aberrations. The same results have also been reported by Bará *et al.*[35]

According to Eq. (10), we can establish a practical criterion of selection to correct or not correct particular aberrations, as follows:

In the presence of a fixed rotation α, the two terms $Z_n^{\pm m}$ should be corrected only if

$$m\alpha < 60°. \quad \text{(R1)}$$

Otherwise, the correction will cause a performance poorer than that with the terms uncorrected. Note that this rule does not prevent the correction of the higher orders but prevents only the correction of some terms in each order: those with high values of m. It is important to remark that this rule is independent of the amount of aberration. As soon as the decentration exceeds the limit value, the correction of the term generates more aberration than the term itself.

With translation, the higher orders generate a larger number of lower-order aberrations, and, moreover, each matrix element depends on the factor $[(n + 1)(n' + 1)]^{1/2}$. We obtain a similar result to that obtained for rotation, i.e., that the higher-order corrections are more sensitive (less tolerant) to translation. It should be noted that not every term of the same order produces the same amount of aberrations. For instance, terms with $m = n$ are less sensitive to translation (although more sensitive to rotation). By calculating the variance of the residual WA, we can also get a practical rule to select the Zernike terms to be corrected after translation. In this case, In the presence of a fixed translation A, the term Z_n^m should be corrected only if

$$A^2 F < 1, \tag{R2}$$

where the factor F is

$$F = (n + 1)\sum_j (n - 2j)\Lambda_j. \tag{11}$$

In Eq. (11),

$$\Lambda_j = \begin{cases} 3 & \text{if } m = 1 \\ 1 & \text{if } m = n - 2j \\ 0 & \text{if } m > n - 2j \\ 2 & \text{otherwise} \end{cases}$$

are nonnull values derived from matrix [T], and the index j takes the integer values from 0 to $(n - 3)/2$. If the translation is in the vertical direction, one should change A by B, and 3 by 1 in the values of Λ_j. The same rule is valid for the correction of the term Z_n^{-m} by changing A by B. Table 3 lists the values of the factor F up to sixth-order.

Table 3. Values of the Factor *F* for Fixed or Gaussian Translations

		F	
n	m	Fixed Translation	Gaussian Translation
3	1	36	48
	3	12	24
4	0	40	80
	2	40	80
	4	20	40
5	1	144	192
	3	78	156
	5	30	60
6	0	140	280
	2	140	280
	4	112	224
	6	42	84

B. Selection Rule to Correct in the Population with Average Decentrations

The rules of selections (R1) and (R2) defined above are to be applied with fixed rotations or translations. However, it would also be useful to have a criterion to perform selective correction that is valid for an expected distribution of decentrations of the correcting method in the average population. By averaging the variance of the residual WA with the Gaussian distribution of Eq. (9) we obtained an expression integrated by individual contributions of each aberration and the consequent next rule:

In the presence of a Gaussian distribution of translation and rotation characterized by σ_t and σ_r, the two terms $Z_n^{\pm m}$ (or the term Z_n^0) should be corrected in the population only if

$$m^2\sigma_r^2 + \frac{\sigma_t^2}{r_0^2}F < 1, \tag{R3}$$

where the factor F is obtained from Eq. (11) with $\Lambda_j = 4$ (2 if $m = n - 2j$, 0 if $m > n - 2j$), σ_t is the width of translation, σ_r is the width of rotation in radians,[41] and r_0 is the pupil radius. Note that the selection depends on the pupil size, and it is more restrictive when the pupil decreases. Values of the factor F are listed in Table 3. Again, it should be noted that the rule is independent of the value of the aberration term. If the limit is exceeded, the term should not be corrected.

Table 4 contains the maximum tolerances, obtained

Table 4. Maximum Decentrations to Have No Benefit (or a 50% Reduction) in the Contribution of the Terms $Z_n^{\pm m}$ to the rms

		No Benefit				50% Reduction	
		$\sigma_t = 0$	$\sigma_r = 0°$	$\sigma_r = 5°$	$\sigma_r = 10°$		
n	m	σ_r	σ_t/r_0	σ_t/r_0	σ_t/r_0	σ_r	σ_t/r_0
2	0	—	—	—	—	—	—
	2	34°	—	—	—	15°	—
3	1	67°	0.144	0.144	0.142	30°	0.072
	3	22°	0.204	0.197	0.176	10°	0.102
4	0	—	0.112	0.112	0.112	—	0.056
	2	34°	0.112	0.110	0.105	15°	0.056
	4	17°	0.158	0.148	0.119	7°	0.079
5	1	67°	0.072	0.072	0.071	30°	0.036
	3	23°	0.080	0.077	0.069	10°	0.040
	5	13°	0.129	0.117	0.078	6°	0.065
6	0	—	0.060	0.060	0.060	—	0.030
	2	34°	0.060	0.060	0.056	15°	0.030
	4	17°	0.067	0.063	0.050	7°	0.034
	6	11°	0.109	0.094	0.043	5°	0.055

from rule (R3), of the distribution of rotation and translation to have no benefit when the terms $Z_n^{\pm m,0}$ are corrected. The table also shows the maximum decentration tolerated to warrant a 50% reduction in the contribution to the rms of these terms. The values in the table provide a measure of the sensitivity to rotation and translation of the different aberration terms and may be used to figure out which terms should be corrected in the presence of a certain distribution of movement. As two examples, (1) for a pupil 6 mm in diameter, the terms Z_6^0 and $Z_6^{\pm 2}$ should not be corrected for a distribution of translation with $\sigma_t > 0.18$ mm, and (2) the correction of the orders higher than fourth gives no benefit if $\sigma_r > 10°$ and $\sigma_t > 0.22$ mm.

These rules may have an important effect on applications. For example, the distribution of movement of contact lenses is empirically known. These data can be used to select a set of aberrations to be corrected: the ones that, despite decentrations, still yield a benefit.

4. RESULTS IN THE POPULATION

A. Effect of Translation and Rotation on Image Quality

The MTF after the higher-order correction with an ICM declines slowly with rotation and translation. We first considered in Fig. 3 the effect of fixed displacements and rotations, although we are ultimately interested in the average performance within ranges of movement. Figure 3 shows the monochromatic MTF for different amounts of rotations and translations averaged across the ten subjects. The MTFs have been averaged for translations along the $\pm X$ and the $\pm Y$ axis, and rotations $\pm \alpha$. The image quality does not fall below the Rayleigh limit for tolerances[42] (Strehl ratio = 0.8) until the displacement is at least 0.1 mm or the rotation angle is 3°. The tolerance progressively increases when the residual aberrations are larger. The ICM would offer no better image quality than an uncorrected eye for a rotation of 45° or a translation of 1 mm and no improvement over a lens that corrects only defocus and astigmatism for a 17° rotation or a 0.6-mm translation. The MTF for a conventional correction shown in Fig. 3 represents a best case because it corresponds to a centered correction.

Maximum translations and rotations of around 0.6 mm and 6° have been reported for soft contact lenses,[30] induced by blinking, and a mean translation of 0.4 mm for a 30° down gaze.[43] Measurements of the motion of the eye during the laser surgery treatment yield a standard deviation of approximately 0.1 mm.[29] These values allow us to estimate a typical distribution of movement of an ideal correcting method to be $\sigma_t = 0.2\text{–}0.3$ mm for translation, and $\sigma_r = 2\text{–}3°$ for rotation. Figure 4 shows the impact of these movements on the average image quality across subjects both in monochromatic and in white light. The MTF has been averaged for each subject with a Gaussian distribution having a width equal to the standard deviations for typical decentrations. In monochromatic light, the average MTF is approximately 2.5 times higher, averaged across spatial frequencies from 0 to 60 c/deg, than the MTF for a lens that corrects only defocus and astigmatism. In white light when the eye's chromatic aberration is not corrected, the polychromatic MTF is, for the same movements, a mean of 1.5–2 times higher than that for a lens that corrects defocus and astigmatism alone. In white light, the tolerance is much larger, since the eye's chromatic aberration is a factor that is dominant over the decentration.

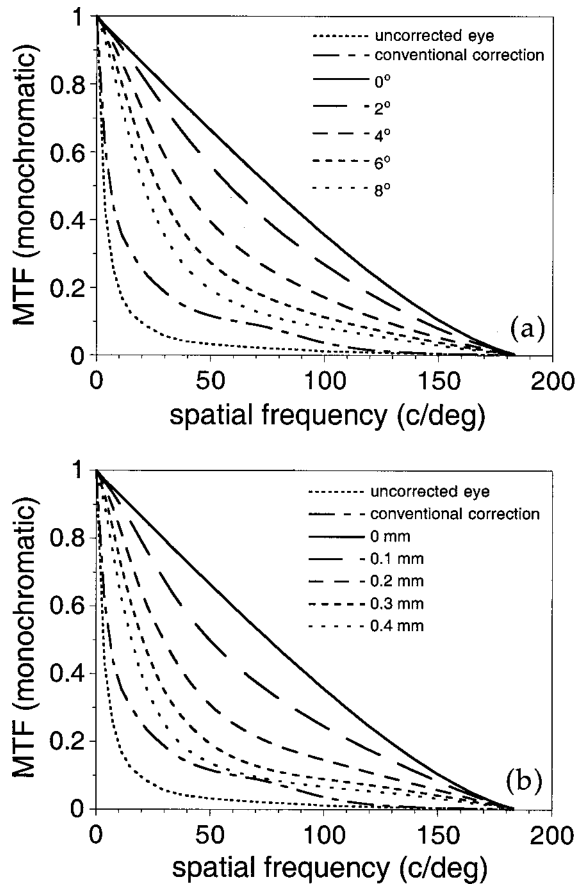

Fig. 3. Average MTF (monochromatic light) across the population of ten eyes after the correction of the higher-order aberrations of each eye with an ideal correcting method, as a function of (a) fixed rotations and (b) translations. Results are averaged across $\pm x$ and $\pm y$ axis for translation and $\pm \alpha$ for rotation. The pupil is 6 mm in diameter. Also shown: the MTF of the average uncorrected eye, and the eye corrected for defocus and astigmatism (conventional correction). The conventional correction is considered perfectly centered.

B. Effect of Decentration on Different Aberration Orders: Selective Correction

Figure 5 plots the mean value across the population of the rms of the residual WA as a function of translation and rotation. Different degrees of correction with the ICM have been considered (up to second, third, fourth, fifth, and sixth order). Thus, for example, in the figure an ICM designed to compensate for the aberrations up to fourth order leaves uncorrected the ocular aberrations of orders higher than fourth, and there is a residual rms even with perfect centration that is due to the incomplete correction [see Eq. (7)]. As we mentioned in Section 3, the higher-order aberrations are less tolerant to translation and rotation. While a perfectly centered ICM progressively compensates for the aberrations when it is designed to correct more orders, the benefit of correcting additional orders decreases when the decentration increases. There is even a loss when the decentration is large.

Figure 6 shows the residual rms averaged with a Gaussian distribution within an interval of movement combining rotation and translation simultaneously. Two conditions are shown: The total correction up to sixth order and a selective correction according to the rule (R3) outlined in Section 3. For each interval of movement, an optimum set of terms was obtained to minimize the residual rms. As an example, we detail here three cases (see Table 4 with $r_0 = 3\,\mathrm{mm}$): (1) for ($\sigma_t = 0.1\,\mathrm{mm}$, $\sigma_r = 5°$) no term must be removed from the optimum correction; (2) for ($\sigma_t = 0.3\,\mathrm{mm}$, $\sigma_r = 5°$) a set including aberrations up to fourth order plus the $Z_5^{\pm5}$ and $Z_6^{\pm6}$ terms is selected; and (3) for ($\sigma_t = 0.3\,\mathrm{mm}$, $\sigma_r = 10°$), only a correction up to fourth order is used. The optimum rms tends to the value corresponding to a second-order correction (0.33 μm; 0.35 and 0.40 μm for $\sigma_r = 5°$ and 10°) when the amount of translation increases.

From Fig. 5 and 6 one can conclude that for typical decentrations it is not worth performing the total correction. In general, the correction of the aberrations only up to fourth or fifth order would yield the same benefit. For large amounts of decentration the total correction produces a loss instead of a benefit over the conventional correction of the second order. From these results it follows that the selective correction is optimum in two senses:

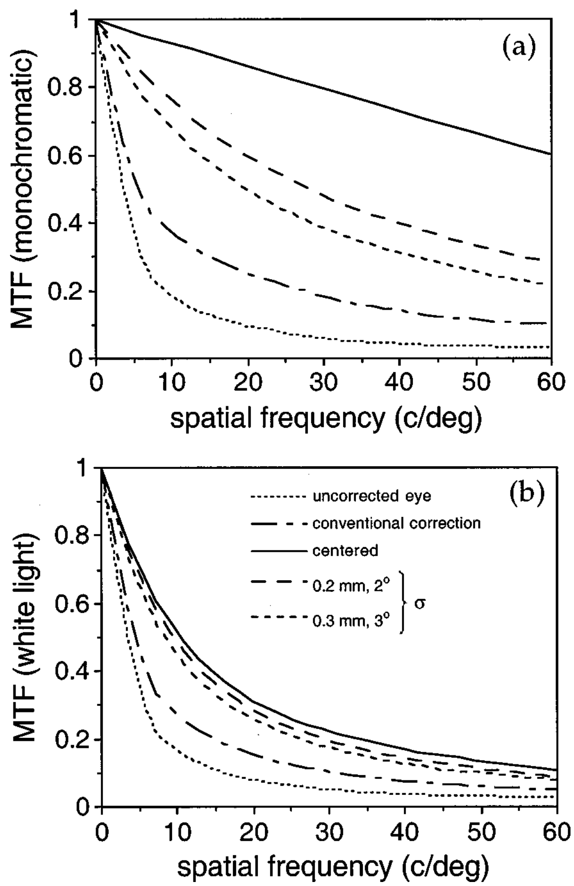

Fig. 4. Average MTF's in the population for typical distributions of movement of the ideal correcting method in (a) monochromatic light and (b) white light. The MTF's were averaged for each subject for a normally distributed translation and rotation with widths of 0.2 mm, 2°; 0.3 mm, 3°. Also shown: the average MTF for the uncorrected eye and the MTF for a centered conventional correction for a 6-mm pupil.

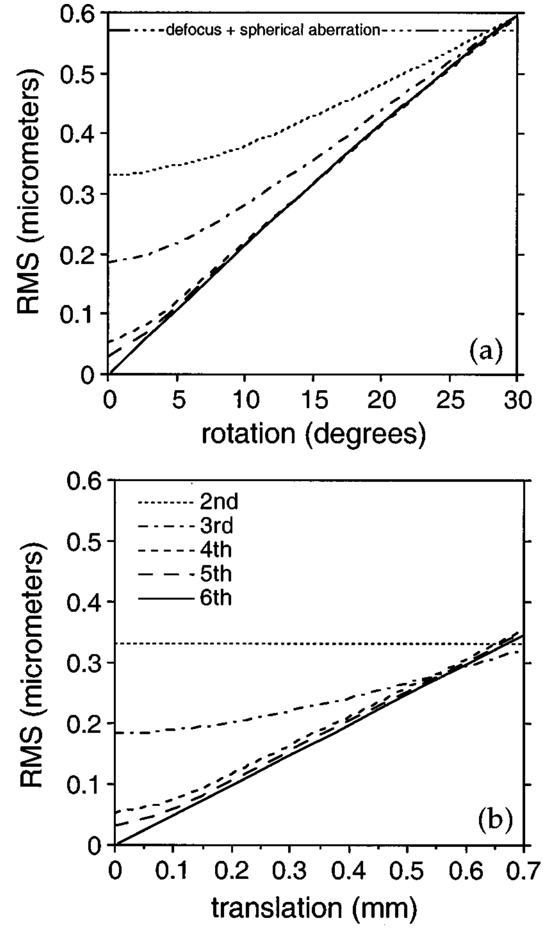

Fig. 5. Rms (mean value in the ten-eye population) of the residual WA for a 6-mm pupil as a function of (a) fixed rotations and (b) translations, when the ideal correcting method corrects the higher-order aberrations up to second-, third-, fourth-, fifth- and sixth-order. Results for translation are averaged across $\pm x$ and $\pm y$ axis. Also shown in (a), the rms when only defocus and spherical aberration are corrected.

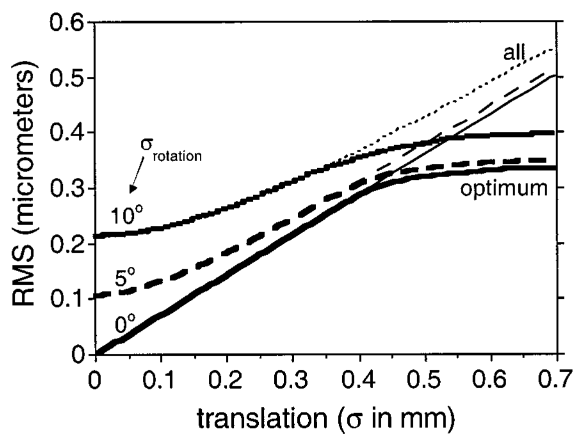

Fig. 6. Rms (mean value in the 10-eye population) of the residual WA for a 6-mm pupil as a function of the width of the Gaussian distribution of movement, with correction of all six orders (thin curves), and optimum correction (thick curves) from the selection rule (R3). The selective correction has been computed for each interval of movement.

First, it is not necessary to implement a total correction, with the possible saving in effort or cost; second, the selective correction may offer even better performance.

5. EXPERIMENTAL STUDY WITH AN ADAPTIVE OPTICS SYSTEM

A. Apparatus

We used an AO system to correct the higher-order aberrations and then measure the loss of compensation produced by decentrations. The AO system (details can be found in Refs. 25 and 44) includes a Shack–Hartmann wave-front sensor for measuring the WA and a 37-actuator deformable mirror (Xinetics, Inc.) for compensating for the measured WA. The mirror lies in a plane conjugate with the eye's pupil. The data from the Shack–Hartmann sensor are transformed by a computer into signals to control the deformable mirror in a closed-loop feedback. The correction with the mirror approaches that of an ICM.

B. Wave-Aberration Data and Measurements

The WA for a subject presenting early keratoconus was measured first. From these data, an artificial eye was made from Plexiglas (using diamond-point turning with a Variform lathe) to simulate the real eye. The eye had a large amount of third-order aberrations in addition to astigmatism. The Zernike coefficients assessed with the Shack–Hartmann wave-front sensor for a 6-mm pupil were as follows: 0.38 and 1.11 μm for horizontal and diagonal astigmatism, -0.93 and 0.09 μm for horizontal and vertical coma, and -0.06 and -0.29 μm for horizontal and vertical triangular astigmatism. The rms was 1.53 μm and 0.98 μm when astigmatism was not included.

Two different experiments were performed. In the first experiment the artificial eye was positioned at the pupil plane by means of a three-dimensional micrometric stage. The assembly allowed the alignment of the eye as well as the application of a known rotation or translation to produce a mismatch between the eye and the deformable mirror. After the compensation was achieved (rms $< 0.01\,\mu$m) with the eye well centered, the residual WA for a 6-mm pupil was measured for rotations of ± 1, ± 3, ± 6, ± 10, ± 15, and $\pm 20°$, and vertical and horizontal translations of 0.1, 0.2, 0.3, and 0.4 mm. We repeated the experiment by correcting with the mirror only the first Zernike terms, and the results were compared with those rendered with the total correction.

In the second experiment, the aberrations of the real eye of the keratoconic subject were corrected with the AO system. The subject's head was then displaced horizontally by different amounts with respect to the centered position.

C. Results

Figure 7 shows the correlation between the Zernike coefficients calculated theoretically and the coefficients measured with the Shack–Hartmann sensor after correction of the aberrations of the artificial eye and application of different rotations and translations. Theoretical and experimental values correlate quite well ($r = 0.98$), and the discrepancies are within experimental error. Figure 8 shows the decrease of the MTF with rotation and translation. The results from the measured WA agree with those obtained from the residual WA calculated theoretically.

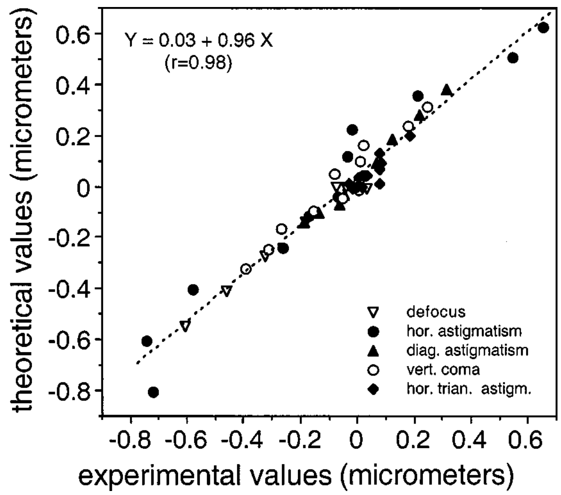

Fig. 7. Theoretical values for the Zernike coefficients of the artificial eye versus experimental values measured for different rotations and translations after correction with adaptive optics.

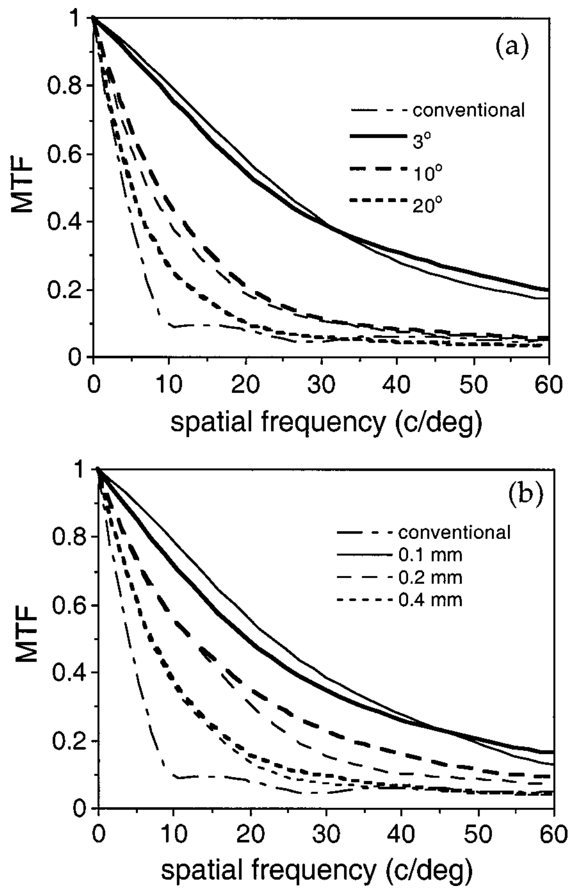

Fig. 8. MTF's for the artificial eye after correcting with adaptive optics and applying different (a) rotations and (b) translations. Comparison is between experimental results (thick curves) and theoretical results (thin curves). Also shown: the MTF for a conventional centered correction.

Although the decline of image quality shown in Fig. 8 with decentration is important, the MTF is still much better than the MTF of the uncorrected eye or the eye corrected for astigmatism, even for large decentrations. For example, with a rotation of 10° or a translation of 0.4 mm, the rms is 0.45–0.55 μm, whereas the rms for the uncorrected eye was 1.53 μm (or 0.98 μm, when the astigmatism was corrected).

We also measured the residual aberrations when the two terms corresponding to triangular astigmatism ($Z_3^{\pm 3}$) were left uncorrected with the AO system. In Fig. 9 we compare the MTF's with the total and the partial correction for different amounts of rotation. When the rotation increases, the differences between the results obtained with the two corrections become smaller. In particular, for a 20° rotation the two curves are equal, in good agreement with rule of selection (R1).

Figure 10 shows the results in the keratoconic subject. The MTF obtained after the best correction is achieved with the deformable mirror (solid curve) is not diffraction limited, indicating that the correction of the higher-order aberrations of this eye was not perfect, even with a good centration. This is because the correcting device is limited by 37 actuators and a maximum stroke per actuator. Moreover, this eye was highly aberrated. The important result here is that the improvement over the conventional correction achieved by the AO correction remains despite decentrations. After a translation of 0.3 mm, the MTF is still much better than that when only second order was corrected.

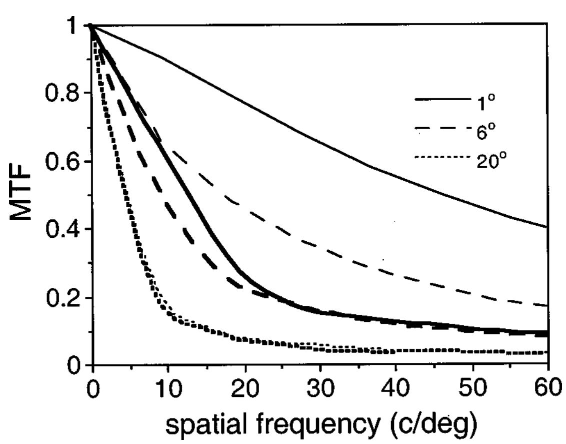

Fig. 9. MTF's for the artificial eye for different rotations (thin curves) in comparison with the MTF's obtained by correcting only astigmatism and coma (thick curves).

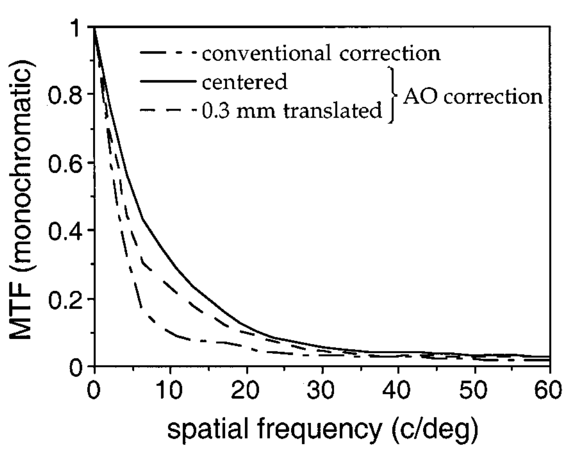

Fig. 10. MTF's for the keratoconic eye after the best correction with adaptive optics for a well-centered position (solid curve) and for a 0.3-mm horizontal translation (dashed curve). The MTF for the conventional correction is also shown for comparison.

6. CONCLUSIONS AND DISCUSSION

Decentration should slightly reduce the optical benefits expected with an ideal correcting method, such as customized contact lenses, adaptive optics, or customized laser refractive surgery. An accurate centration is required to reach the diffraction limit. However, for typical movements the correcting method could still achieve a benefit relatively larger than that obtained with conventional lenses that correct defocus and astigmatism alone. In monochromatic light, a normal population, and large pupils, benefits in modulation transfer between twofold and fourfold may be obtained compared with the second-order correction despite decentrations. In white light, chromatic aberration dominates, and therefore the benefit obtained when the higher-order monochromatic aberrations are corrected is more modest. Nevertheless, an improvement in modulation approximately 1.5 and 2 times that obtained with the correction of defocus and astigmatism alone may still be achieved in white light. Moreover, in that case the tolerance to translation and rotation is much larger than in monochromatic light, and the effect of decentration on the optical performance of the ideal correcting method in broadband illumination should be a minor limitation.

The effects of both fixed decentrations and movement have been examined in this paper. Fixed decentrations may appear as a result of a disagreement between the pupil axis and the position of equilibrium of a contact lens on the cornea or as a result of a difference between the axis used to measure the aberrations and the actual axis of a customized lens or that used to apply a laser procedure. Figure 3 shows the impact of these fixed decentrations. Unless the decentrations are very large, the correcting method still provides better performance than a standard correction. In addition, fixed decentrations can be reduced if they are systematic and detectable. For instance, a systematic inferior-temporal decentration of ~0.4 mm of some soft contact lenses has been reported.[45] This problem could be solved with a customized contact lens that incorporates the expected decentration. Also, inferior decentrations of the eye up to 0.25 mm have been measured during laser surgery.[29] In this case, tracking techniques may help to apply the treatment with the correct centration. More important could be the effect of variable decentrations caused, for example, by blinking, rotations of the eye when gazing, or rapid movements of the eye. We explored this issue by considering a Gaussian movement of the correcting method within a certain interval. Figure 4 shows that the effect of these movements on image quality is expected to be slight. Owing to the typical values for rotation and translation, the rotation seems to be a less important factor than translation.

Several factors not considered here could affect the performance of a real correcting method. An important caveat is that the process of removing tissue in refractive surgery is a departure from the ideal correcting, phase-plate model we were working with. For example, displacing a correction for defocus only does introduce other aberrations in refractive surgery but not in the ideal correcting case we considered. Therefore these results should be taken as a first approximation in the case of laser refractive surgery. A specific study of this problem will be published elsewhere. In the case of customized contact lenses, a possible bending of the lens when it conforms to the cornea or a change in the corneal surface induced by contact will change the predicted coupling among contact lens and eye aberrations. However, measurements suggests that the aberrations of soft contact lenses and those of the eye sum without much alteration of one by the other.[46] In any case, this would not affect our model if the process for choosing an ideal correcting lens is to place a diagnostic spherical lens on the eye and then measure the wave-front aberration through the combination of eye plus contact lens, adding the higher-order aberrations to the next and final lens. The only difference would be with respect to a conventional sphero-cylindrical correction. If a conventional lens is no longer spherical or toric, higher-order aberrations such as coma will appear in addition to the simple shift in the retinal image. This could explain the fact that subjects wearing conventional spherical contact lenses have lower visual performance than those wearing spectacles. Nevertheless, our analysis rests on the comparison of higher-order correction with conventional correction. We also do not consider the influence of scattered light, which could increase with a contact lens, and changes in the tear film caused by wearing contact lenses. No attempt was made here to include a possible blur caused by the movement of a contact lens as it restores from decentration with the consequent superposition of images differently shifted in time. This should not have important implications on the retinal image, considering the high return speed (1–2 mm/s) of contact lenses.[47,48] On the other hand, we considered the performance of an ideal correcting method based on a static correction. However, the ocular aberrations have temporal variations[49] (owing to several sources, such as fluctuations of the accommodation or small variations of the thickness of the tear film) that further limit the maximum benefit that can be obtained with a static correction, although this will also produce an increase in the tolerance to the aberrations caused by decentration. Finally, we did not consider the neural factors that affect vision. While we assumed that a perfect optical quality would enhance visual performance, this performance probably declines for some specialized visual tasks because of aliasing by photoreceptors.[50]

We have presented matrices that are valid for converting the wave aberration of a correcting method after a rotation or a translation with respect to the axis of the eye and have provided general expressions to explicitly obtain the residual aberrations. The analysis done with the use of Zernike expansions allows us to understand how translation and rotation affect the different aberration terms. Rotation produces a residual aberration of the same kind as the aberration corrected, in an amount that depends on the angular order of the aberration. On the other hand, translation generates lower-order aberrations below the order corrected, by a factor that depends on the radial order. These facts indicate that the higher-order corrections are more sensitive to translation and rotation. Thus it may not be worth correcting certain aberrations, since doing so could offer no benefit at all and might even produce more residual aberrations than without correction. Our equations showed which aberrations are more beneficial to correct. Depending on the magnitude of the decentration, even larger benefits could be achieved by correcting an adequate set of terms instead of all of them. We have given rules for selecting aberrations that should be included for an optimum correction in an ideal correcting method that must tolerate decentrations. Selective corrections may be optimized for every subject, depending on the particular aberrations. As a general conclusion in the normal population, a practical correction for tolerating typical decentrations could be carried out with a correcting method that includes only the aberrations up to fourth order.[51]

In addition to the clinical interest of these theoretical results, the equations derived are useful in other applications. For instance, sometimes it is necessary to convert the aberrations measured relative to a reference axis to another axis. The conversion matrices of rotation and translation allow us to reobtain the Zernike coefficients of the wave aberration for a change of axis. Also, these matrices suggest the possibility of making new generations of plates for generating aberrations. A pair of plates with opposite profiles of spherical aberration has been already used to generate coma aberration by means of a relative displacement[52] that extends the method of Alvarez,[53] which generates focus and astigmatism by means of two phase plates with a cubic profile. As an example, from matrix $[T]$ in Table 2 we could make plates including an aberration of fifth order to generate spherical aberration, defocus, and astigmatism. This may be of interest for the fabrication of instruments for calibrating wave-front-sensor devices.

Bará *et al.*[35] have also theoretically studied the loss of compensation suffered when a well-matched ocular correcting phase plate[22] is displaced or rotated. Although Bará *et al.* used the rms to measure the loss of performance in monochromatic light, we also studied the effect of decentration on the MTF (in both monochromatic and white light). The relationship between rms in the pupil plane and retinal image is not straightforward. The MTF more clearly quantifies retinal image quality, which is what visual performance ultimately depends on. On the other hand, we studied the combined effect of rotation and translation, as well as the performance with dynamic decentrations in addition to fixed decentrations. Also, our paper extends the work of Bará *et al.* by showing the effect of decentration on different aberration orders and comparing the expected benefit from a correction that includes higher orders with a conventional correction of defocus and astigmatism alone.

Bará *et al.* defined the degree of compensation as the unity minus the ratio between the rms, after compensation and the original rms of the uncorrected eye. We can use our values of rms from Fig. 5 as a confirmatory process for the correction of all the orders and the rmss for the uncorrected eyes to calculate the degree of compensation. We obtain compensation of 0.7–0.9, averaged across subjects, for rotations of 5–15°, while Bará *et al.* obtain 6–18°, which is in quite good agreement. Regarding translation, we find that a 0.5-mm translation produces on average a 0.75 compensation degree, although Bará *et al.* obtain a value of ~0.5. This difference is not surprising if one considers that the results are for two different samples of subjects (ten eyes in our case and three eyes in the Bará *et al.* paper). The pupil sizes are also different (6 mm for us, 6.5 mm for Bará *et al.*). But probably the main difference is how the wave front is treated outside the eye's pupil. Whereas we consider that the correcting method extends beyond the eye's pupil, these authors assume, in a worst-case situation, that the correcting plate compensates only the aberrations in a region equal to the eye's pupil. Thus after a translation there is a pupil region that remains uncorrected, which decreases the degree of compensation. This can explain why the two sets of results agree for rotations but differ for translations, our rms being somewhat higher than theirs for a given translation. As we have also shown here, Bará *et al.* found that the most critical factor is translation rather than rotation or axial displacement.

The experimental results obtained in the artificial eye with the adaptive optics system confirmed the theoretical estimations and showed how a big benefit could be achieved with customized higher-order correction in subjects with large amounts of aberrations, such as keratoconics. We concluded from this study that an ideal correcting method customized for correcting higher-order aberrations seems promising, despite decentrations. However, to be effective such a method should correct more than a single aberration (spherical aberration or coma, for instance),[52,54,55] in addition to defocus and astigmatism, although it may not include the highest orders that one can measure because of unfavorable effects with decentrations.

ACKNOWLEDGMENTS

This work was supported by the National Institutes of Health (grants EY04367, EY01319), The National Science Foundation Science and Technology Center for Adaptive Optics (grant 5-24182) managed by the University of California at Santa Cruz (cooperative agreement AST-9876783, and a research contract from Bausch & Lomb (Rochester, New York). A. Guirao is grateful for a postdoctoral fellowship from the Ministerio de Educación y Cultura (Spain).

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$$Z_n^{\pm m}(x - \Delta x, y - \Delta y) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \left(\Delta x \frac{\partial}{\partial x} + \Delta y \frac{\partial}{\partial y} \right)^k [Z_n^{\pm m}(x, y)].$$

Thus the matrix $[T]$ can be constructed by taking the first, second, etc., derivatives of the Zernike polynomials. [For the first derivatives, see R. J. Noll, "Zernike polynomials and atmospheric turbulence," J. Opt. Soc. Am. **66**, 207–211 (1976)].

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